

# OPTIMAL RECOVERY OF FUNCTIONS FROM HARDY-SOBOLEV CLASSES

OSIPENKO K. YU. (MOSCOW)

Let  $X$  be a normed linear space of functions defined on some set  $E$ . Suppose that  $B$  is a subspace of  $X$  and  $l_1, \dots, l_n \in B^*$ . Set

$$\begin{aligned} i_n(B, X) &:= \inf_{l_1, \dots, l_n \in B^*} \inf_{S: \mathbb{R}^n \rightarrow X} \sup_{\|f\|_B \leq 1} \|f - S(l_1 f, \dots, l_n f)\|_X, \\ s_n(B, X) &:= \inf_{t_1, \dots, t_n \in E} \inf_{S: \mathbb{R}^n \rightarrow X} \sup_{\|f\|_B \leq 1} \|f - S(f(t_1), \dots, f(t_n))\|_X. \end{aligned} \quad (1)$$

Any functionals for which the infimum in (1) is attained we shall call optimal functionals. The values  $i_n(B, X)$  and  $s_n(B, X)$  were introduced by S. Fisher and C. Micchelli [1].

We study these quantities for the Hardy-Sobolev class  $\tilde{H}_{\infty, \beta}^r$  which is the set of all  $2\pi$ -periodic, real on the real axis and analytic in the strip  $S_\beta := \{z : |\operatorname{Im} z| < \beta\}$  functions such that  $|f^{(r)}(z)| \leq 1$ ,  $z \in S_\beta$ . We show that  $i_n(\tilde{H}_{\infty, \beta}^r, C)$  coincides with the Kolmogorov, linear and Gel'fand  $n$ -widths of  $\tilde{H}_{\infty, \beta}^r$  in  $C$ . Moreover, we prove that Fourier coefficients  $\{a_j(f)\}_{j=0}^k$ ,  $\{b_j(f)\}_{j=1}^k$  are optimal linear functionals in problem (1) for  $n = 2k - 1, 2k$ .

For  $r = 0$  and even  $n$  we also prove that evaluations of  $f \in \tilde{H}_{\infty, \beta}$  in the system of equidistant points from  $[0, 2\pi)$  are optimal functionals, too. That is

$$i_{2k}(\tilde{H}_{\infty, \beta}, C) = s_{2k}(\tilde{H}_{\infty, \beta}, C).$$

This equality does not valid for odd  $n$ . We show that

$$i_{2k-1}(\tilde{H}_{\infty, \beta}, C) < s_{2k-1}(\tilde{H}_{\infty, \beta}, C).$$

## REFERENCES

1. S. D. Fisher and C. A. Micchelli, *Optimal sampling of holomorphic functions. II*, Math. Ann. **273** (1985), no. 1, 131–147.
2. K. Yu. Osipenko, *Exact  $n$ -widths of Hardy-Sobolev classes*, Constr. Approx. (to appear).