The Hadamard three-circle theorem states that if \( f(z) \) is a holomorphic function on the annulus \( r_1 \leq |z| \leq r_2 \) and \( M(r) = \max_{|z|=r} |f(z)| \), then \( \log M(r) \) is a convex function of the \( \log r \). The conclusion of the theorem can be restated as

\[
M(r) \leq M(r_1)^{\frac{\log r_2/r}{\log r_2/r_1}} M(r_2)^{\frac{\log r/r_1}{\log r_2/r_1}}
\]

for any three concentric circles of radii \( r_1 < r < r_2 \).

The Hardy-Littlewood-Polya inequality is the following one

\[
\| x^{(k)}(\cdot) \|_{L^2(\mathbb{R})} \leq \| x(\cdot) \|_{L^2(\mathbb{R})}^{1-k} \| x^{(r)}(\cdot) \|_{L^2(\mathbb{R})}^{\frac{k}{2}}.
\]

One may formulate it in the Hadamard three-circle theorem form. Namely, in the following form. \( \log \| x^{(k)}(\cdot) \|_{L^2(\mathbb{R})} \) is a convex function of \( k \). We use this fact to obtain optimal recovery methods for the \( k \)-th derivative on the basis of inaccurate information about some other derivatives.