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direction in the Radon domain. Over the decades, the numerous results including the sparsity model based approach has enabled the reconstruction of the image inside the region of interest (ROI) from the limited knowledge of the data. However, unlike these existing methods, we try to reconstruct the entire CT image from the limited knowledge of the sinogram via the tight frame regularization and the simultaneous sinogram extrapolation. Our proposed model shows more promising numerical simulation results compared with the existing sparsity model based approach.

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Time: June 28(Tue) 3:30–3:55, Room: ECC B157

MS06-02. **Various methods for ill-posed inversion in Quantitative Susceptibility Mapping**

Joint work with Tian Liu, Weiwei Chen, and Yi Wang

Shuai Wang, School of Electronic Engineering, University of Electronic Science and Technology of China

This talk is concerned with the well-known ill-posed dipole inversion problem of Quantitative Susceptibility Mapping (QSM) in Magnetic Resonance Imaging (MRI). We systematically analyzed the inversion part of QSM in terms of their treatment of noise in the data fidelity term and their choice of prior. Based on analysis of the effects of noise treatment in the data fidelity term and the choice of prior in QSM based on a few published methods in literature, it was found that noise whitening and structure priors are useful in improving QSM using regularized minimization. This is a joint work with Tian Liu, Weiwei Chen, Yi Wang.

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Time: June 28(Tue) 4:00–4:25, Room: ECC B157

MS06-03. **Intensity Nonuniformity Correction Method in brain MR images**

Joint work with Hemant Tagare

Yunho Kim, Dept. of Mathematical Sciences, UNIST

Intensity nonuniformity artifact is unavoidable in MR imaging due to RF coil inhomogeneity, gradient-driven eddy current, interactions within the body, etc., which result in nonuniform signals where we expect uniform ones. Especially, we expect three values in T1 weighted brain MR images for GM (gray matter), WM (white matter), CSF (cerebrospinal fluid). However, the measured data contain smooth variation in values over the region. In this talk, we will describe the problem in detail and propose a rigorous mathematical framework, where we can detect the shape of smooth variation in values revealing uniform underlying signal. The only assumption of our model is that the artifact has a slowly varying characteristic over a bounded region, which is enough tofactor out the artifact correctly.

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Time: June 28(Tue) 4:30–4:55, Room: ECC B157

MS06-04. **The optimal methods for inversion of the noisy k-plane transform**

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In many applied and theoretical problems one needs to recover a function (functional or operator) from the information, which can be incomplete or given with an error. Such problems are investigated in the optimal recovery theory - a modern branch of approximation theory. In general the problem is to find the best approximation of a value of linear operator $U : X \to Z$ on a given set $X$ from values of another linear operator $I : X \to Y$ (called information) given with an error in some metric. In present work we consider the $k$-plane transform – an operator, that maps a function on Rd to the set of its integrals over all $k$-planes. This operator is widely used in the computerized tomography theory, which deals with the numerical reconstruction of functions from their linear integrals. Special cases are the Radon transform ($k = d - 1$) and the X-ray transform ($k = 1$). For the particular classes of functions there exist different inversion formulas that allow to produce exact reconstruction. We consider the case when the $k$-plane transform is measured with an error $\delta > 0$ in the mean square metric. Consider $G_{k,d}$ the Grassmann manifold of (non-oriented) $k$-dimensional subspaces in $\mathbf{R}^d$. Given representation of a point $x \in \mathbf{R}^d$ in a form $x = x' + x''$, $x' \in \pi$, $x'' \in \pi^\perp$, $\in G_{k,d}$ the $k$-plane transform is defined by the integral along the plane parallel to through the point $x''$:

$$Pf(\pi, x'') = P_\pi f(x'') = \int_\pi f(x' + x'')dx', \; x'' \in \pi^\perp.$$

Its domain is the manifold of all $k$-planes in $\mathbf{R}^d$ $\mathcal{G}_{k,d} = (\pi, x'') : \pi \in G_{k,d}, x'' \in \pi^\perp$. We will work with the class of functions

$$W = \{f \in L_2(\mathbf{R}^d) : \|\Lambda^\alpha f\|_{L_2(\mathbf{R}^d)} \leq 1; \quad Pf \in L_2(\mathcal{G}_{k,d})\}.$$

The Lambda operator is defined for $\alpha > 0$ by the equation $\widehat{\Lambda^\alpha f}(\xi) = |\xi|\alpha \widehat{f}(\xi)$ on the set of functions $f \in L_2(\mathbf{R}^d)$ that satisfy the condition $|\xi|\alpha \in f(\xi) \in L_2(\mathbf{R}^d)$. Suppose that for every function $f \in W$ we know function $g \in L_2(\mathcal{G}_{k,d})$ such that $\|Pf - g\|_{L_2(\mathcal{G}_{k,d})} \leq \delta$. On this information we want to recover function $\Lambda^\beta$ as an element of $L_2(\mathbf{R}^d)$, where $0 \leq \beta < \alpha$. We consider all possible *methods* or *recovery* – arbitrary maps $m : L_2(\mathcal{G}_{k,d}) \to L_2(\mathbf{R}^d)$. For every method of recovery $m$ define its error $e(\delta, m)$ by

$$e(\delta, m) = \sup_{\substack{f \in W, g \in L_2(\mathcal{G}_{k,d}) \\ \|Pf - g\|_{L_2(\mathcal{G}_{k,d})} \leq \delta}} \|\Lambda^\beta f - m(g)\|_{L_2(\mathbf{R}^d)}.$$

The smallest error among all the methods is called the *error* of the optimal recovery

$$E(\delta) = \inf_{m : L_2(\mathcal{G}_{k,d}) \to L_2(\mathbf{R}^d)} e(\delta, m).$$

Method $m$ for which the error of the optimal recovery is attained, i.e. $e(\delta, m) = E(\delta)$, is called optimal. We present the explicit construction for the optimal methods and the error of the optimal recovery. As a consequence, we give one inequality for the norms of the degree of the Lambda operator and the $k$-plane transform. Particular cases include new inversion methods and inequalities for the classical Radon and X-ray transforms.

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Time: June 28(Tue) 5:00–5:25, Room: ECC B157

MS06-05. **Towards Beam Hardening Correction in X-ray CT**

Joint work with Hao Gao, Sung Min Lee, and Jin Keun Seo

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