OPTIMAL RECOVERY IN HARDY–SOBOLEV SPACES AND AN ANALOGUE OF SPLINES FOR ANALYTIC FUNCTIONS

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Denote by H^r_∞ the class of analytic in the unit disk D functions f for which $|f^{(r)}(z)| \le 1$, $z \in D$. We consider the problem of optimal recovery of $f(\tau)$, $\tau \in (-1,1)$, using the information $If = (f(t_1),\dots,f(t_{n+r}))$, $t_j \in (-1,1)$. We calculate the value

$$e(\tau, H^r_\infty, I) = \inf_{S\colon \mathbb{C}^{n+r}\to\mathbb{C}} \sup_{f\in H^r_\infty} |f(\tau) - S(If)|$$

and find an optimal algorithm S_0 for which the infimum is attained.

For every system of points $-1 < t_1 < \ldots < t_{n+r} < 1$ there exist such $-1 < x_1 < \ldots < x_n < 1$ that the function

$$g \in X_{n+r} = \operatorname{span}\{1, z, \dots, z^{r-1}, g_1(z), \dots, g_n(z)\},$$

where

$$g_j(z) = \int_0^z \frac{(z-t)^{r-1}(1-t^2)\omega_j(t)}{(r-1)!(1-x_j t)^2}\,dt, \quad \omega_j(t) = \prod_{\substack{k=1\\ k\neq j}}^{n} \frac{t-x_k}{1-x_k t},$$

which interpolates f at the points $t_1,\dots,t_{n+r}$ gives an optimal method of recovery $f(\tau) \approx g(\tau)$ for all $\tau \in (-1,1)$. Thus the space X_{n+r} is an analogue of polynomial splines which appear in the similar problem for the Sobolev classes.