

Optimal Recovery, Best Approximation, and Extremum Theory

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The talk is devoted to the problems of optimal recovery of linear functionals and operators by incomplete and/or inaccurate information and their interconnections with the problems of analysis and approximation theory. The last several years have seen the growing interest to the problems where it is necessary to recover function values, or values of its derivatives at some points, or value of integral of a function, or to recover a function itself in some metric and so on. Usually in this situation we have some information (incomplete and/or inaccurate) about a function: for example, we know function values (may be with some errors) at some points, or some of the Fourier coefficients, or moments, or initial data if this function is a solution of differential equation and so on. The problem is to use all this information in the best way for recovering of one or another functional or operator from this function. The statement of optimal recovery problem is going back conceptually to the papers of Kolmogorov of the 30s of the past century where the problem of best approximation method on a class of functions was stated.

In the talk a unique approach to investigation of optimal recovery problems based on general principles of extremum theory and convex duality is given. A series of examples will be presented where this approach allows to obtain explicit expressions for optimal methods of recovery. In these problems there are some interesting effects connected with excessive information of input data which is apparently important in practical applications. Moreover, it will be shown that there are close connections between optimal recovery theory and problems of analysis and approximation theory (polynomials least deviating from zero, problem of moments, inequalities for derivatives of polynomials and smooth functions, approximation of analytic functions).

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