OPTIMAL RECOVERY OF ANALYTIC FUNCTIONS FROM THEIR FOURIER COEFFICIENTS GIVEN WITH AN ERROR

K. YU. OSIPENKO (MOSCOW)

Let H be a Hilbert space and $\{e_j\}$ a complete orthonormal system in H. We consider the problem of optimal recovery of the linear functional (x,f), $f\in H$, from approximate values of Fourier coefficients $x_j=(x,e_j)$.

Put

$$e_n(f,\delta):=\inf_{\varphi\colon\,\mathbb{C}^n\to\mathbb{C}}\ \sup_{\substack{x\in H\\ \|x\|\le 1}}\ \sup_{\substack{\widetilde{x}_j,\ j=1,\dots,n\\ |\widetilde{x}_j-x_j|\le\delta_j}}|(x,f)-\varphi(\widetilde{x}_1,\dots,\widetilde{x}_n)|.$$

Set

$$a_+:=\begin{cases}a, & a\ge 0,\\ 0, & a<0.\end{cases}$$

Theorem. *The method*

$$(x,f)\approx\sum_{j=1}^{n}(1-\lambda\delta_j|f_j|^{-1})_+\overline{f}_j\widetilde{x}_j$$

is an optimal method of recovery and

$$e_n(f,\delta)=\lambda+\sum_{j=1}^{n}\delta_j(|f_j|-\lambda\delta_j)_+,$$

where $\lambda\in(0,\|f\|]$ *is a solution of the equation*

$$\|f\|^2-\sum_{j=1}^{n}(|f_j|^2-\lambda\delta_j^2)_+-\lambda^2=0.$$

Using this Theorem we construct optimal methods of recovery of 2π-periodic and analytic in a strip functions and its derivatives from approximate values of their Fourier coefficients.