n-WIDTHS AND KOLMOGOROV'S INEQUALITY IN HARDY–SOBOLEV CLASSES

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Let $H^r_{\infty,\beta}$, $r \ge 0$ be the class of all real-valued on $\mathbb{R}$ and analytic on the strip $S_\beta := \{\, z \in \mathbb{C} : |\operatorname{Im} z| < \beta \,\}$ functions which satisfy the restriction $|f^{(r)}(z)| \le 1$, $z \in S_\beta$. Denote by $\widetilde{H}^r_{\infty,\beta}$ 2π-periodic functions from $H^r_{\infty,\beta}$. Let Λ and Λ' be the complete elliptic integrals of the first kind with moduli λ and $\lambda' = \sqrt{1-\lambda^2}$. For every $\nu \in (0,\infty)$ determine $\lambda = \lambda(\nu)$ by the equation

$$\frac{\Lambda'}{\Lambda} = \frac{4\beta\nu}{\pi}.$$

For $r \ge -1$ put

$$\Phi^\beta_{\nu,r}(z) := \frac{\pi}{\sqrt{\lambda}\Lambda\nu^r} \sum_{s=0}^{\infty} \frac{\sin((2s+1)\nu z - \pi r/2)}{(2s+1)^r \sinh((2s+1)2\nu\beta)}.$$

The functions $\Phi^\beta_{\nu,r}$ play a role which is analogous to that played by the Euler perfect splines in the case of the Sobolev classes of functions. It can be shown that

$$\|\Phi^\beta_{\nu,r}\|_\infty := \frac{\pi}{\sqrt{\lambda}\Lambda\nu^r} \sum_{s=0}^{\infty} \frac{(-1)^{s(r+1)}}{(2s+1)^r \sinh((2s+1)2\nu\beta)}$$

where $\|\cdot\|_\infty$ is the norm in $L_\infty(\mathbb{R})$.

Denote by d_n, λ_n and d^n the Kolmogorov, linear and Gel'fand n-widths, respectively. The following two results are considered.

Theorem 1. *For all integer $r \ge 0$ and $L_\infty := L_\infty[0,2\pi]$*

$$d_{2n}\big(\widetilde{H}^r_{\infty,\beta}, L_\infty\big) = \lambda_{2n}\big(\widetilde{H}^r_{\infty,\beta}, L_\infty\big) = d^{2n}\big(\widetilde{H}^r_{\infty,\beta}, L_\infty\big) = \|\Phi^\beta_{n,r}\|_\infty.$$

Theorem 2. *Suppose that $\delta \in (0,\infty)$ if $r \ge 1$, and $\delta \in (0,1)$ if $r = 0$. Then for all $1 \le k \le r+1$*

$$\sup_{\substack{f \in H^r_{\infty,\beta} \\ \|f\|_\infty \le \delta}} \|f^{(k)}\|_\infty = \|\Phi^\beta_{\nu,r-k}\|_\infty$$

where ν is determined by the equation

$$\|\Phi^\beta_{\nu,r}\|_\infty = \delta.$$