OPTIMAL RECOVERY OF FUNCTIONS IN $H_{2,\beta}$

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Let $S_\beta := \{z \in \mathbb{C} : |\operatorname{Im} z| < \beta\}$ be a strip in the complex plane. Denote by $H_{2,\beta}$ the class of 2π-periodic, analytic in S_β functions f, which satisfy

$$\sup_{0\le\eta<\beta} \frac{1}{4\pi}\int_0^{2\pi} (|f(t+i\eta)|^2 + |f(t-i\eta)|^2)\, dt \le 1.$$

Set

$$s_n(H_{2,\beta}, C(\mathbb{T})) := \inf_{t_1,\dots,t_n\in\mathbb{T}} \inf_{A\colon \mathbb{C}^n\to C(\mathbb{T})} \sup_{f\in H_{2,\beta}} \|f - A(f(t_1),\dots,f(t_n))\|_{C(\mathbb{T})},$$

$$i_n(H_{2,\beta}, C(\mathbb{T})) := \inf_{l_1,\dots,l_n} \inf_{A\colon \mathbb{C}^n\to C(\mathbb{T})} \sup_{f\in H_{2,\beta}} \|f - A(l_1 f,\dots,l_n f)\|_{C(\mathbb{T})},$$

where $l_1,\dots,l_n$ are linear continuous functionals. The values $i_n(H_{2,\beta}, C(\mathbb{T}))$ are coincide with linear and Gel'fand n-widths which were calculated in [1]. We study the values $s_n(H_{2,\beta}, C(\mathbb{T}))$ and compare them with $i_n(H_{2,\beta}, C(\mathbb{T}))$.

Denote by K and K' be the complete elliptic integrals of the first kind with moduli k and $k' = \sqrt{1-k^2}$. Suppose that k is defined by the equation

$$\frac{\pi K'}{2K} = \beta.$$

We prove that

$$\frac{s_{2n-1}(H_{2,\beta}, C(\mathbb{T}))}{i_{2n-1}(H_{2,\beta}, C(\mathbb{T}))} = 2\sqrt{\frac{kK}{\pi}\sinh\beta} + O(e^{-4\beta n}),$$

$$\frac{s_{2n}(H_{2,\beta}, C(\mathbb{T}))}{i_{2n}(H_{2,\beta}, C(\mathbb{T}))} = 2\sqrt{\frac{K}{\pi}\tanh\beta} + O(e^{-4\beta n}).$$

References

[1] K. Yu. Osipenko. On n-widths of holomorphic functions of several variables. J. Approx. Theory **82**, 1995, 135–155.

[2] K. Yu. Osipenko and K. Wilderotter. Optimal information for approximating periodic analytic functions. Math. Comput. (to appear).