ON OPTIMAL RECOVERY OF PERIODIC ANALYTIC FUNCTIONS

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Let $S_\beta := \{z \in \mathbb{C} : |\operatorname{Im} z| < \beta\}$ be a strip in the complex plane. Denote by H_p^β the class of 2π-periodic, analytic in S_β functions f, which satisfy

$$\sup_{0\le\eta<\beta} \frac{1}{4\pi}\int_{\mathbb{T}} (|f(t+i\eta)|^p + |f(t-i\eta)|^p)\,dt \le 1, \quad 1 \le p < \infty,$$

$$\sup_{z\in S_\beta} |f(z)| \le 1, \quad p = \infty.$$

We consider the problem of optimal recovery of $Lf = f(\xi)$ or $f'(\xi)$, $\xi \in \mathbb{T}$, using the information $If = (f(x_1), \dots, f(x_n))$, $x_j \in \mathbb{T}$. We calculate the intrinsic error

$$e(L, H_p^\beta, I) := \inf_{A:\mathbb{C}^n\to\mathbb{C}} \sup_{f\in H_p^\beta} |Lf - A(If)| \tag{1}$$

and find an optimal algorithm A^* for which the infimum in (1) is attained.

For example, if $Lf = f'(0)$ and $If = (f(-h), f(h))$, then an optimal algorithm is given by

$$f'(0) \approx \frac{K}{\pi} \operatorname{dn}^{\frac{2(p-1)}{p}} \frac{K}{\pi}h \; \frac{f(h) - f(-h)}{\operatorname{sn} \dfrac{2K}{\pi}h},$$

where $\operatorname{dn} z$ and $\operatorname{sn} z$ are the Jacobi elliptic functions with modulus k defined by the equation

$$\frac{\pi K'}{2K} = \beta$$

(K and K' are the complete elliptic integrals of the first kind with moduli k and $k' = \sqrt{1-k^2}$).