OPTIMAL RECOVERY OF FUNCTIONS FROM HARDY–SOBOLEV CLASSES

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Let X be a normed linear space of functions defined on some set E. Suppose that B is a subspace of X and $l_1, \dots, l_n \in B^*$. Set

$$(1)\quad \begin{aligned} i_n(B,X) &:= \inf_{l_1,\dots,l_n \in B^*} \inf_{S\colon \mathbb{R}^n \to X} \sup_{\|f\|_B \le 1} \|f - S(l_1 f, \dots, l_n f)\|_X, \\ s_n(B,X) &:= \inf_{t_1,\dots,t_n \in E} \inf_{S\colon \mathbb{R}^n \to X} \sup_{\|f\|_B \le 1} \|f - S(f(t_1), \dots, f(t_n))\|_X. \end{aligned}$$

Any functionals for which the infimum in (1) is attained we shall call optimal functionals. The values $i_n(B,X)$ and $s_n(B,X)$ were introduced by S. Fisher and C. Micchelli [1].

We study these quantities for the Hardy–Sobolev class $\widetilde{H}^r_{\infty,\beta}$ which is the set of all 2π-periodic, real on the real axis and analytic in the strip $S_\beta := \{z : |\operatorname{Im} z| < \beta\}$ functions such that $|f^{(r)}(z)| \le 1$, $z \in S_\beta$. We show that $i_n(\widetilde{H}^r_{\infty,\beta}, C)$ coincides with the Kolmogorov, linear and Gel'fand n-widths of $\widetilde{H}^r_{\infty,\beta}$ in C. Moreover, we prove that Fourier coefficients $\{a_j(f)\}_{j=0}^k$, $\{b_j(f)\}_{j=1}^k$ are optimal linear functionals in problem (1) for $n = 2k-1, 2k$.

For $r = 0$ and even n we also prove that evaluations of $f \in \widetilde{H}_{\infty,\beta}$ in the system of equidistant points from $[0, 2\pi)$ are optimal functionals, too. That is

$$i_{2k}(\widetilde{H}_{\infty,\beta}, C) = s_{2k}(\widetilde{H}_{\infty,\beta}, C).$$

This equality does not valid for odd n. We show that

$$i_{2k-1}(\widetilde{H}_{\infty,\beta}, C) < s_{2k-1}(\widetilde{H}_{\infty,\beta}, C).$$

References

[1] S. D. Fisher and C. A. Micchelli, Optimal sampling of holomorphic functions, II, *Math. Ann.* **273** (1985), 131–147.

[2] K. Yu. Osipenko, Exact n-widths of Hardy–Sobolev classes, *Constr. Approx.* (to appear).