APPROXIMATION OF ANALYTIC FUNCTIONS

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We consider the so-called Hardy–Sobolev classes $\widetilde{H}^r_{\infty,\beta}$ which are the sets of all 2π-periodic and analytic in the strip $S_\beta := \{z : |\operatorname{Im} z| < \beta\}$ functions such that

$$|f^{(r)}(z)| \le 1, \quad z \in S_\beta.$$

When $r = 0$ these classes are known as the Hardy classes. For these classes we study the following three themes:

1. Optimal interpolation.
2. Optimal qudratures.
3. n-Widths.

All these problems were studied very intensively for smooth functions, especially for the Sobolev classes. By the efforts of many mathematicians a general theory was built for the classes which can be represented as a convolution with cyclic variation diminishing or totally positive kernels.

This general theory can not be applied to the classes of analytic functions because as usual they are not represented in such form. Methods which are usually used for smooth functions are based on the calculation of sign changes of functions. The number of sign changes is found with the help of Rolle's theorem. But Rolle's theorem is not valid for complex-valued functions. It is not even clear what will replace such notion as sign changes in the complex case.

Nevertheless several results which are very close to the smooth case can be obtained in the analytic case. Moreover, sometimes a general theory for smooth and analytic cases can be built. As a consequence of this theory we determine exact values of n-widths both for the Sobolev and Hardy–Sobolev classes $\widetilde{H}^{r,\mathbb{R}}_{\infty,\beta}$ which are the sets of all functions from $\widetilde{H}^r_{\infty,\beta}$ that are real on the real axis.

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