ON OPTIMAL RECOVERY OF DERIVATIVES OF ANALYTIC FUNCTIONS

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Let W be a class of sufficiently smooth real-valued or complex-valued functions. We study the problem of optimal recovery of the k-th derivative of $f \in W$ at some fixed point ξ on the basis of information If where I is a linear operator from a linear space $X \supset W$ into a normed linear space Y. In general this information can be given with some error. The value

$$e_k(\xi, W, I, \delta) := \inf_{S : Y \to \mathbb{R}(\mathbb{C})} \sup_{\substack{f \in W \\ \|y - If\|_Y \le \delta}} |f^{(k)}(\xi) - S(y)|$$

is called the intrinsic error of optimal recovery. Any function S for which the infimum is attained is called an optimal method of recovery.

We consider this problem for the Hardy classes and Hardy–Sobolev classes of functions defined in the unit disk or in the strip $\{z \in \mathbb{C} : |\operatorname{Im} z| < \beta\}$.

If W is a convex and balanced set, then by the duality

$$e_k(\xi, W, I, \delta) = \sup_{\substack{f \in W \\ \|If\|_Y \le \delta}} |f^{(k)}(\xi)|.$$

For the Hardy class H_∞ one of the first result relating to this extremal problem was obtained by J. Dieudonné in 1931. He proved that

$$\sup_{\substack{|f(z)| \le 1,\ |z| < 1 \\ f(0) = 0}} |f'(\xi)| = \begin{cases} 1, & |\xi| \le \sqrt{2} - 1, \\ \dfrac{1 + |\xi|^2}{4|\xi|(1 - |\xi|^2)}, & |\xi| > \sqrt{2} - 1. \end{cases}$$

That is in the case when $k = 1$, $If = f(0)$, and $\delta = 0$ the unit disk divided in two parts in which extremal functions have different forms.

We study how this property transforms in general cases. We obtain the intrinsic error and optimal algorithms for the periodic Hardy classes.

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